

**Exercise 38**

Find the derivative of the function.

$$y = \sqrt{1 + xe^{-2x}}$$

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**Solution**

Take the derivative using the chain rule.

$$\begin{aligned} y' &= \frac{dy}{dx} = \frac{d}{dx} (1 + xe^{-2x})^{1/2} \\ &= \frac{1}{2}(1 + xe^{-2x})^{-1/2} \cdot \frac{d}{dx}(1 + xe^{-2x}) \\ &= \frac{1}{2}(1 + xe^{-2x})^{-1/2} \cdot \frac{d}{dx}(xe^{-2x}) \\ &= \frac{1}{2}(1 + xe^{-2x})^{-1/2} \cdot \left\{ \left[ \frac{d}{dx}(x) \right] e^{-2x} + x \left[ \frac{d}{dx}(e^{-2x}) \right] \right\} \\ &= \frac{1}{2}(1 + xe^{-2x})^{-1/2} \cdot \left\{ (1)e^{-2x} + x \left[ (e^{-2x}) \cdot \frac{d}{dx}(-2x) \right] \right\} \\ &= \frac{1}{2}(1 + xe^{-2x})^{-1/2} \cdot \{ (1)e^{-2x} + x [(e^{-2x}) \cdot (-2)] \} \\ &= \frac{1}{2}(1 + xe^{-2x})^{-1/2} \cdot (1 - 2x)e^{-2x} \\ &= \frac{(1 - 2x)e^{-2x}}{2\sqrt{1 + xe^{-2x}}} \end{aligned}$$